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Slew Maneuver of a Flexible Space Structure with Constraint on Bending Moment

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A control scheme is proposed for a rest-to-rest slew maneuver of a flexible space structure with constraints on the maximum value of the bending moment of the flexible structure. The flexible space structure treated is a rigid body equipped with a flexible appendage. The slew maneuver is a rest-to-rest maneuver with a constraint on the bending moment at the root of the flexible appendage, and a control scheme is studied to minimize the bending moment due to the structural vibration inevitably excited by the slew maneuver of the flexible structure. The optimal control profile is studied both analytically and experimentally to solve the control algorithm and also to verify the validity of the implementation. Results of the numerical and experimental analyses of the present control are compared with those of the well-known time-optimal control and the robust time-optimal control to demonstrate the effectiveness of the present control scheme.

I. Introduction

As construction technology is developed for space systems, their characteristic dimensions are expected to increase. Large space structures (LSS) must be light in weight to offset huge launch costs; thus, their structure subsystem components will also be highly flexible.

An extremely small amount of damping in the flexible structures is expected due to the vacuum feature of the space environment, and thus the vibration of the flexible space structure will be difficult to suppress once it has been excited. Also, it is widely known that large flexible space structural modes are characterized by many low-frequency vibrational modes, and control/structural interaction associated issues, as well as observation (measurement) spillover, may be of concern.

Extensive studies of the minimum and near-minimum time slewing of large flexible space structures have been reported.[1–7] The resulting open-loop time-optimal control obtained as a switching bang–bang type is usually very sensitive to system modeling errors, in many cases, because the maneuver time t_f is chosen as the performance index in this control scheme and the system modeling error is not directly considered. Large flexible space structural modes are associated with various uncertain system parameters, which make it difficult to predict the dynamics. To overcome this problem, a robust time-optimal control method has been proposed and applied to

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flexible space structures.[8–10] In the robustified method, constraints for robustness with respect to structural frequency uncertainty are added to the time-optimal problem as the final conditions, and the resulting robustfied, time-optimal solution for the flexible space structure is obtained as a multiswitch bang–bang control. The input shaping control method is another method to improve the robustness of the time-optimal control algorithm and has been applied to flexible space structures.[11–14] Singhose et al.[15] have proposed a method to minimize maneuver time subject to constraints on residual vibration magnitudes, sensitivity to modeling errors, rest-to-rest slew distance, and the transient deflection amplitude.

These time-optimal control algorithms are studied with the main objective being the suppression of residual vibrations, without consideration of the bending moment induced on the flexible appendage during the maneuvers. Note that the value of the bending moment can be representative of the vibrational feature of the flexible appendage. Also note that flexible motions inevitably excited during slew maneuvers could be a serious drawback to the achievement of stringent high performance of LSS with very low bending rigidity and damping characteristics.

The present paper focuses on the behavior of the bending moment induced on the flexible appendage during the transitional motions of a flexible space structure and introduces the concept of minimum bending-moment control. The time integral of the square of the bending moment is chosen as a performance index in the present concept. Large structural deflections induce large internal loads on flexible structures, and the limitation on the deflection during the maneuver is an important feature. The minimum bending-moment control with inequality constraints on the maximum value of the bending moment is studied. The final time in the minimum bending-moment control is set as close to that resulting from the time-optimal control because the final time also remains as one of the most important performance standards. Performance of the present control is compared with that of the time-optimal control and that of the robust time-optimal control by both numerical simulations and experimental analysis.

The minimum bending-moment control problem is studied by the nonlinear programming method, which is useful for solving the optimal control problem with inequality constraints on the state variables.

The model employed in the experiment is a rigid-body equipped with a flexible beam running on a rail. All control algorithms have been implemented in experiments to verify the validity of the analytical results obtained from theory.

II. Problem Formulation

A. System Model

Figure 1 shows a model of a space structure consisting of a rigid body equipped with a flexible appendage. The appendage is assumed to be a cantilever beam with one end fixed to the rigid body and the other end free. Translational and vibrational motions are assumed to be planer two-dimensional motion. The control input is assumed to be applied only on the rigid body. When the structural damping

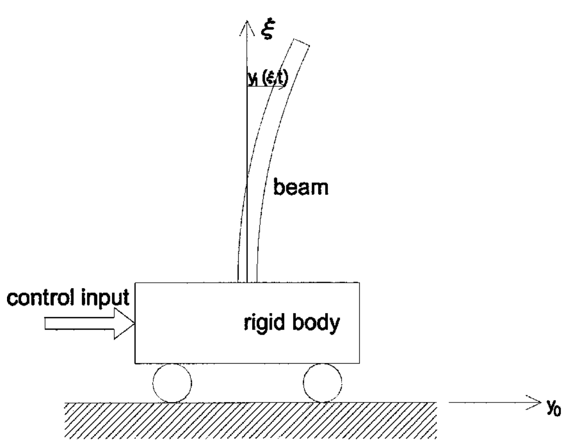

Fig. 1 System model.

and the air drag are neglected, the behavior of the present model is described by the following equations of motion:

$$(m_0 + \rho l)\ddot{y}_0(t) + \rho \int_0^l \frac{\partial^2 y_1(\xi, t)}{\partial t^2}\,\mathrm{d}\xi = u(t) \tag{1}$$

$$\rho \ddot{y}_0(t) + \rho \frac{\partial^2 y_1(\xi, t)}{\partial t^2} + EI \frac{\partial^4 y_1(\xi, t)}{\partial \xi^4} = 0 \tag{2}$$

with the boundary conditions

$$y_1 = \frac{\partial y_1}{\partial \xi} = 0(\xi = 0), \qquad \frac{\partial^2 y_1}{\partial \xi^2} = \frac{\partial^3 y_1}{\partial \xi^3} = 0(\xi = l) \tag{3}$$

where the dot indicates differentiation with respect to time, y_0 is the displacement of the rigid body, u is the control input, and m_0, ρ, and EI are the mass of the rigid body, the mass density of the beam per unit length, and the bending rigidity of the beam, respectively. The parameters ρ and EI are assumed to be constant along the beam. When the flexible vibration of the appendage by the vibrational mode

$$y(\xi, t) = \sum_{i=1}^{n} \Phi_i(\xi) y_i(t)$$

is expanded and the state variable is transformed from $\boldsymbol{y} = [y_0 \ \dot{y}_0 \ y_1 \ \dot{y}_1 \ , \dots , \ y_n \dot{y}_n]^T$ to $\boldsymbol{x} = [x_1 \ x_2 \ , \dots , \ x_{2n+2}]^T$ by using the transformaion $\boldsymbol{x} = T\boldsymbol{y}$, where T is the transformation matix, Eqs. (1) and (2) can be rewritten in the following state-space equation:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}u(t) \tag{4}$$

where the coefficient matrices A and B are

$$\boldsymbol{A} = \mathrm{blockdiag}[A_0 \quad A_1 \quad \cdots \quad A_n] \tag{5}$$

$$\boldsymbol{B} = [0 \quad \phi_0 \quad 0 \quad \phi_1 \quad \cdots \quad \phi_n]^T \tag{6}$$

$$A_0 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad A_i = \begin{bmatrix} 0 & 1 \\ -\omega_i^2 & 0 \end{bmatrix} \tag{7}$$

A_0 is the matrix associated with the rigid-body motions, A_i is the ith modal matrix, ϕ_i is the ith modal gain, and ω_i is the ith flexible modal frequency. In the present study, we assume that the following constraint exists on the control input:

$$|u(t)| \leq 1 \qquad \text{or} \qquad 1 - u^2(t) \geq 0 \tag{8}$$

B. Minimum Bending-Moment Control

The minimum bending-moment control concept introduced in this paper describes a method to find the control input that minimizes the following performance index in the rest-to-rest slew maneuver:

$$J = \int_{t_0}^{t_f} M^2(t)\,\mathrm{d}t \tag{9}$$

subject to Eqs. (4) and (8), and

$$\boldsymbol{x}(t_0) = [0 \quad 0 \quad 0 \quad \cdots \quad 0]^T \tag{10a}$$

$$\boldsymbol{x}(t_f) = [x_{1f} \quad 0 \quad 0 \quad \cdots \quad 0]^T \tag{10b}$$

where x_{1f} is the objective final position of the rigid body and $M(t)$ is the bending moment at the root of the beam. Let us assume that the vibrational motion of the flexible appendage under consideration includes only the first bending mode of vibration to simplify the analysis. The bending moment at the root of the beam can then be expressed as

$$M(t) = \alpha x_3(t) \tag{11}$$

where α is a coefficient determined from the bending stiffness and the length of the beam.

III. Nonlinear Programming Problem

A. Performance Index

Integrating Eq. (4), we have

$$x(t) = e^{A(t-t_0)}x(t_0) + \int_{t_0}^{t_f} e^{A(t-\tau)}Bu(\tau)\,d\tau \tag{12}$$

Using the condition that $x(t_0) = 0$ in the present formulation of the rest-to-rest maneuver, this equation is seen to be equivalent to the following equations:

$$x_1(t) = \int_{t_0}^{t} \phi_0(t_f - \tau)u(\tau)\,d\tau \tag{13a}$$

$$x_2(t) = \int_{t_0}^{t} \phi_0 u(\tau)\,d\tau \tag{13b}$$

$$x_3(t) = \int_{t_0}^{t} \frac{\phi_1}{\omega}\sin\omega(t-\tau)u(\tau)\,d\tau \tag{13c}$$

$$x_4(t) = \int_{t_0}^{t} \phi_1\cos\omega(t-\tau)u(\tau)\,d\tau \tag{13d}$$

Substituting Eq. (13c) into Eq. (9), we obtain

$$J = \int_{t_0}^{t_f} \alpha^2\left[\int_{t_0}^{t}\frac{\phi_1}{\omega}\sin\omega(t-\tau)u(\tau)\,d\tau\right]^2 dt \tag{14}$$

B. Necessary Conditions

When the constraints on the state variables at the final time, Eq. (10b), and the control input constraint, Eq. (8), are added to the performance index, the Lagrangian to be minimized is defined as follows:

$$\begin{aligned} L(u,\lambda) = &\int_{t_0}^{t_f}\left[\int_0^t \frac{\alpha\phi_1}{\omega}\sin\omega(t-\tau)u(\tau)\,d\tau\right]^2 dt \\ &+ \nu_1\left\{\int_{t_0}^{t_f}\phi_0(t_f-\tau)u(\tau)\,d\tau - x_{1f}\right\} \\ &+ \nu_2\int_{t_0}^{t_f}\phi_0 u(\tau)\,d\tau + \nu_3\int_{t_0}^{t_f}\frac{\phi_1}{\omega}\sin\omega(t-\tau)u(\tau)\,d\tau \\ &+ \nu_4\int_{t_0}^{t_f}\phi_1\cos\omega(t-\tau)u(\tau)\,d\tau + \int_{t_0}^{t_f}\lambda(t)[1-u^2(t)]\,dt \end{aligned} \tag{15}$$

where

$$\lambda(t)\begin{cases} = 0 & |u| < 1 \\ \geq 0 & |u| = 1 \end{cases} \tag{16}$$

Taking the Fréchet derivative of Eq. (15) with respect to u, we obtain

$$\begin{aligned} \frac{L(u+\epsilon\eta) - L(u)}{\epsilon} = &\nu_1\int_{t_0}^{t_f}\phi_0(t_f-\tau)\eta(\tau)\,d\tau \\ &+ \nu_2\int_{t_0}^{t_f}\phi_0\eta(\tau)\,d\tau + \nu_3\int_{t_0}^{t_f}\frac{\phi_1}{\omega}\sin\omega(t_f-\tau)\eta(\tau)\,d\tau \\ &+ \nu_4\int_{t_0}^{t_f}\phi_1\cos\omega(t_f-\tau)\eta(\tau)\,d\tau \\ &+ 2\int_{t_0}^{t_f}\left[\int_{t_0}^{t}\left(\frac{\alpha\phi_1}{\omega}\right)^2\sin\omega(t-\tau)u(\tau)\,d\tau\right] \\ &\times\left[\int_{t_0}^{t}\sin\omega(t_f-\tau)\eta(\tau)\,d\tau\right]dt - 2\int_{t_0}^{t_f}\lambda(t)u(t)\eta(t)\,dt \end{aligned} \tag{17}$$

The stationary condition of Eq. (15) is that Eq. (17) reduces to zero for arbitrary η; thus, we can obtain the integral equation of the optimal control input. Note that, when the optimal control input u^* satisfies $|u^*| = 1$, the optimal control input has been already obtained and that it is not necessary to solve the integral equation. The integral equation is

$$\begin{aligned} &\int_{t_0}^{t_f}\left\{\nu_1\phi_0(t_f-\tau) + \nu_2\phi_0 + \frac{\nu_3\phi_1}{\omega}\sin\omega(t_f-\tau) + \nu_4\phi_1\cos\omega(t_f-\tau)\right\}\eta(\tau)\,d\tau \\ &+ \int_{t_0}^{t_f}\left[\int_{t_0}^{t}\left(\frac{\alpha\phi_1}{\omega}\right)^2\sin\omega(t-\tau)u(\tau)\,d\tau\right] \\ &\times\left[\int_{t_0}^{t}\sin\omega(t-\tau)\eta(\tau)\,d\tau\right]dt = 0 \end{aligned} \tag{18}$$

Equation (18) reduces to the following equation through the process of derivation as described in the Appendix:

$$\begin{aligned} \int_{t_0}^{\tau}K_1(\tau,s)u(s)\,ds = &-\int_{t_0}^{t_f}K_2(\tau,s)u(s)\,ds \\ &- \frac{\omega^2}{2\phi_1^2}\left[\nu_1\phi_0(t_f-\tau) + \nu_2\phi_0 + \frac{\nu_3\phi_1}{\omega}\sin\omega(t_f-\tau) + \nu_4\phi_1\cos\omega(t_f-\tau)\right] \end{aligned} \tag{19}$$

where

$$K_1(\tau,s) = [\omega(s-\tau)\cos\omega(s-\tau) - \sin\omega(s-\tau)]/2\omega \tag{20a}$$

$$K_2(\tau,s) = \{-2\omega(s-t_f)\cos\omega(s-\tau) + \sin\omega(s-\tau) - \sin[2\omega t_f - \omega(s+\tau)]\}/4\omega \tag{20b}$$

Note that the integral equation expressed by Eq. (19) is the first kind of Volterra integral equation, which can be converted into the second kind of Volterra integral equation. By differentiating both sides of the equation four times with respect to τ, we get

$$\begin{aligned} &\frac{\omega}{2}\int_{t_0}^{\tau}[-\omega(\tau-s)\cos\omega(\tau-s) - 3\sin\omega(\tau-s)]u(s)\,ds + u(\tau) \\ &= a\cos\omega\tau + b\sin\omega\tau - \frac{\omega^4\nu_4\cos\omega(t_f-\tau) + \omega^3\nu_3\sin\omega(t_f-\tau)}{2\phi_1} \end{aligned} \tag{21}$$

where a and b are constants. Taking the Laplace transform of both sides of Eq. (21) yields

$$\begin{aligned} U(s) = &-\big\{(s^2+\omega^2)\big[-2(as+b\omega)\phi_1 - \omega^6(\nu_3 - s\nu_4)\cos\omega t_f \\ &+ \omega^5\big(s\nu_3 + \omega^2\nu_4\big)\sin\omega t_f\big]\big\}\big/2\omega^2 s^4\phi_1 \end{aligned} \tag{22}$$

Because the denominator and numerator of the right side of Eq. (22) are the fourth power of s and a third-order polynomial of s, respectively, we can obtain a form of the optimal control input by taking the inverse Laplace transform of both sides of preceding equation as follows:

$$u(t) = q_1 + q_2 t + q_3 t^2 + q_4 t^3 = p^T(t)q \tag{23}$$

where

$$p^T(t) = [1 \quad t \quad t^2 \quad t^3], \qquad q^T = [q_1 \quad q_2 \quad q_3 \quad q_4] \tag{24}$$

Let us assume that the optimal control input u^* consists of two parts: $|u^*| = 1$ and $u^* = p^T(t)q$. Parameters q can be determined according to the switching times of the control input and the final states.

C. Determination of the Parameters

A method to determine the parameters $\boldsymbol{q}$ is explained using a simple case in which the time response of the control input can be divided into the following three parts:

$$u(t) = \begin{cases} 1 & 0 \le t \le t_1 \\ q_1 + q_2 t + q_3 t^2 + q_4 t^3 & t_1 \le t \le t_2 \\ -1 & t_2 \le t \le t_f \end{cases} \tag{25}$$

To meet the final condition of the state vector, the optimal control input must satisfy the following equation:

$$\boldsymbol{x}(t_f) = \int_{t_0}^{t_1} e^{A(t_f-\tau)} B \cdot (1)\,\mathrm{d}\tau + \int_{t_1}^{t_2} e^{A(t_f-\tau)} B \cdot \boldsymbol{p}^T(\tau)\,\mathrm{d}\tau \cdot \boldsymbol{q} - \int_{t_2}^{t_f} e^{A(t_f-\tau)} B \cdot (1)\,\mathrm{d}\tau \tag{26}$$

and the parameters $\boldsymbol{q}$ can be obtained by

$$\boldsymbol{q} = \boldsymbol{L}^{-1}\boldsymbol{R} \tag{27}$$

where

$$\boldsymbol{L} = \int_{t_1}^{t_2} e^{A(t_f-\tau)} B \cdot \boldsymbol{p}^T(\tau)\,\mathrm{d}\tau \tag{28}$$

and

$$\boldsymbol{R} = \boldsymbol{x}(t_f) - \int_{t_0}^{t_1} e^{A(t_f-\tau)} B\,\mathrm{d}\tau + \int_{t_2}^{t_f} e^{A(t_f-\tau)} B\,\mathrm{d}\tau \tag{29}$$

To obtain a more general time response for the control input, we can extend the method by varying the number of switchings and the switching time.

IV. Extension to the Problem with an Inequality Constraint on the Value of the Bending Moment

A. Problem Formulation

In this section, we consider the minimum bending-moment control problem associated with the constraint on the maximum value of the bending moment at the root of the flexible beam. The problem is described as follows:

$$\text{minimize } J = \int_{t_0}^{t_f} M^2(t)\,\mathrm{d}t$$

subject to Eqs. (4), (8), (10a), (10b), and a constraint due to the maximum bending moment of the beam, $M_{\max}$,

$$\|M(t)\|_\infty \le M_{\max} \tag{30}$$

where $\| \ \|_\infty$ is the infinity norm.

There are two possibilities for the optimal value of $M(t)$, either $|M(t)| < M_{\max}$ or $|M(t)| = M_{\max}$. The state of the bending moment during maneuvering can be divided into two parts: the constraint is not effective when $|M(t)| < M_{\max}$ and the constraint is effective when $|M(t)| = M_{\max}$. This leads naturally the partition of the maneuvering period into the following five phases.

Phase 1:

$$M(t) < M_{\max} \qquad t \in [0, t_1]$$

Phase 2:

$$M(t) = M_{\max} \qquad t \in [t_1, t_2]$$

Phase 3:

$$|M(t)| < M_{\max} \qquad t \in [t_2, t_3]$$

Phase 4:

$$M(t) = -M_{\max} \qquad t \in [t_3, t_4]$$

Phase 5:

$$M(t) > -M_{\max} \qquad t \in [t_4, t_f]$$

Because we treat the rest-to-rest problem, the time response of the control input should be symmetric with respect to the middle of the maneuvering period. Thus, the control input during phase 4 and phase 5 can be determined using the control input during phase 1 and phase 2.

Let us define the following vector to denote simply the state of the bending moment:

$$\boldsymbol{M}(t) = [M(t)\dot{M}(t)], \qquad \dot{M} = \frac{\mathrm{d}}{\mathrm{d}t}M \tag{31}$$

The state vector of the bending moment during phase 1, $\boldsymbol{M}(t)$, must transfer from the initial state, $\boldsymbol{M}(t_0) = [0, 0]$, to the final state of phase 1, $\boldsymbol{M}(t_1) = [M_{\max}, 0]$, and must satisfy constraint (30). The control input during phase 2 must maintain the value of the bending moment within $M_{\max}$ at any time $t \in [t_1, t_2]$. To realize this state, the control input during phase 2 must be constant and be the same as the final control input of phase 1, $u_1(t_1)$. This is because the inertial force induced by the control input should balance the resistant bending moment of the flexible beam. Similarly, the state vector of the bending moment during phase 3 must transfer from the state $\boldsymbol{M}(t_2)$=$[M_{\max}, 0]$ to the state $\boldsymbol{M}(t_3) = [-M_{\max}, 0]$. In summary, the optimal control input at each phase can be obtained as follows.

Phase 1:

$$u_1^*(t) = q_1 + q_2 t + q_3 t^2 + q_4 t^3 \qquad t \in [0, t_1] \tag{32a}$$

Phase 2:

$$u_2^*(t) = q_1 + q_2 t_1 + q_3 t_1^2 + q_4 t_1^3 \qquad t \in [t_1, t_2] \tag{32b}$$

Phase 3:

$$u_3^*(t) = q_5 + q_6 t + q_7 t^2 + q_8 t^3 \qquad t \in [t_2, t_3] \tag{32c}$$

Phase 4:

$$u_4^*(t) = q_5 + q_6 t_3 + q_7 t_3^2 + q_8 t_3^3 \qquad t \in [t_3, t_4] \tag{32d}$$

Phase 5:

$$u_5^*(t) = -q_1 + q_2(t - t_f) - q_3(t - t_f)^2 + q_4(t - t_f)^3 \qquad t \in [t_4, t_f] \tag{32e}$$

B. Conditions to Determine Parameters

Let us define the following variables to simply explain the conditions to determine the parameters:

$$\boldsymbol{x}'(t) = [x_3(t) \quad x_4(t)]^T, \qquad \boldsymbol{q}_1 = [q_1 \quad q_2 \quad q_3 \quad q_4]^T$$

$$\boldsymbol{q}_2 = [q_5 \quad q_6 \quad q_7 \quad q_8]^T, \qquad \boldsymbol{p} = [1 \quad t \quad t^2 \quad t^3]^T$$

Because there are eight parameters, q_i, $i = 1, \ldots, 8$, to be determined in Eqs. (32a–32e), we choose the following eight conditions in accordance with the control input.

For conditions 1 and 2, the control input during phase 1 transfers the substate vector $\boldsymbol{x}'(t)$ from $\boldsymbol{x}'(t_0) = [0, 0]^T$ to $\boldsymbol{x}'(t_1) = [x_{3\max}, 0]$, where $x_{3\max}$ is the value determined from $M_{\max}$.

For conditions 3 and 4, the control input during phase 2 transfers the substate vector $\boldsymbol{x}'(t)$ from $\boldsymbol{x}'(t_2) = [x_{3\max}, 0]^T$ to $\boldsymbol{x}'(t_3) = [-x_{3\max}, 0]$.

For condition 5, the main body reaches the desired position at $t = t_f$.

For condition 6, the control inputs $u_2(t)$ and $u_3(t)$ are the same at the time $t = t_2$.

For condition 7, the control input u_3 has a point of inflection at $t = t_f/2$. Because the problem treated in this paper is the rest-to-rest maneuver problem, it seems natural that the control input is symmetric with respect to the middle of the maneuver time.

For condition 8, the control input at $t = 0$ is 0. This condition is introduced to avoid an excessive load on the beam at the start.

After prescribing the form of the time response of the bending moment and choosing the conditions for the determination of the parameters, the original problem can be formulated as the optimization problem of the variables t_1, t_2, and t_f, which minimize the performance index.

V. Numerical Simulations and Experiments

A. Experimental Setup

The model of the flexible appendage is a 960-mm-long aluminum beam with a cross section of 25 × 9 mm and is clamped to the rigid body that moves in a horizontal plane through actuation of a linear motor. Figure 2 shows the apparatus of the experimental setup. Because the linear motor employed in this study can not be operated by force control but only by velocity control, the velocity-based control is employed to actuate the rigid body. The procedure to apply the optimal control input to the rigid body is summarized as follows:

1) The maneuvering time is divided into 160 sections.

2) In accord with the optimal solutions, position and velocity of the rigid body to be realized are calculated at each node of the discretized time.

3) The rigid body is actuated by the linear motor.

4) The position of the rigid body is sensed through the incremental encoder, and the velocity of the rigid body is compensated to follow the reference velocity by feeding the torque to the motor according to the difference between the reference velocity and the current velocity.

Two full-bridge strain gauges are located at the root of the beam to measure the bending moment at the root of the beam. A signal from the gauges is amplified and recorded in a processor, NEC PC9801, using a sampling time of 0.003 s. The system parameters for the experiment are listed in Table 1.

B. Minimum Bending-Moment Control Without Inequality Constraint on the Bending Moment

To verify the validity of the minimum bending-moment control, results are compared with the results of the well-known time-optimal control and robust time-optimal control as shown in Figs. 3–6. Figures 3 and 5 show control input of time-optimal control and robust time-optimal control, and Figs. 4 and 6 show the time responses of the bending moment of these control methods. Figures 7 and 8 show the time responses of the control input and the bending moment of the minimum bending-moment control, respectively. The final time for the minimum bending-moment control without inequalty constraint is set to be 2.0 s. The resulting control inputs of the time-optimal control and the robust time-optimal control are multiswitch bang–bang controls. It is known that the number of switchings of the robust time-optimal control is greater than that of the time-optimal control. The residual vibration decreases in the case of the minimum bending-moment control as well as the robust time-optimal control in comparison with the time-optimal control, as shown in Fig. 8.

The switchings of control input are seen to appear shortly after the start and before the end in the time responses of both the robust time-optimal control and the minimum bending-moment control. On the other hand, the switching starts rather late at the beginning and finishes before closing to the end of the time response of the time-optimal control. The maximum value of the bending moment

Table 1 System parameters

Parameter	Value
Weight of the rigid body m_0	14.0 kg
Bending stiffness EI	1.01 N · m
Maximum control input	1.0 N
Beam length l	0.96 m
Unit weight of beam ρ	0.128 kg
Final position of the rigid body x_{1f}	2.0 m

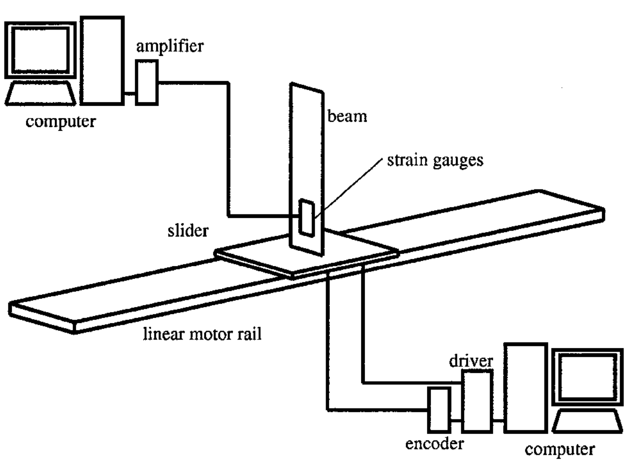

Fig. 2 Experimental setup apparatus.

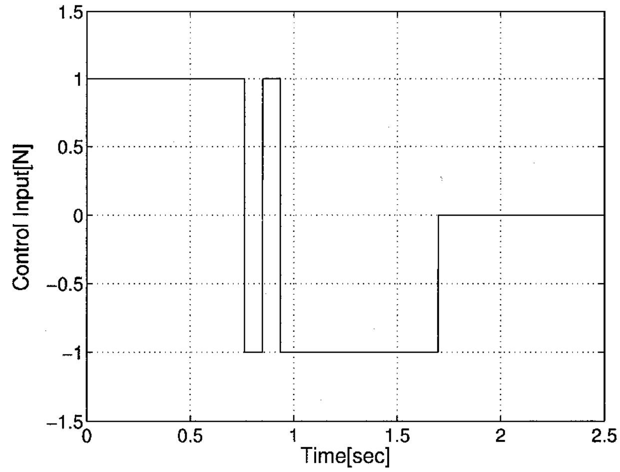

Fig. 3 Time response of control input (time-optimal control).

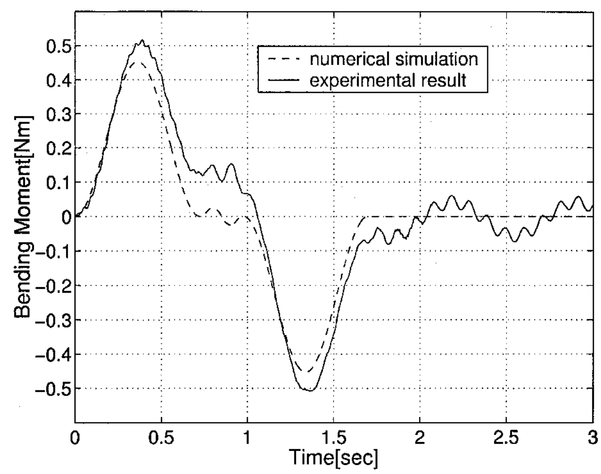

Fig. 4 Time response of bending moment (time-optimal control).

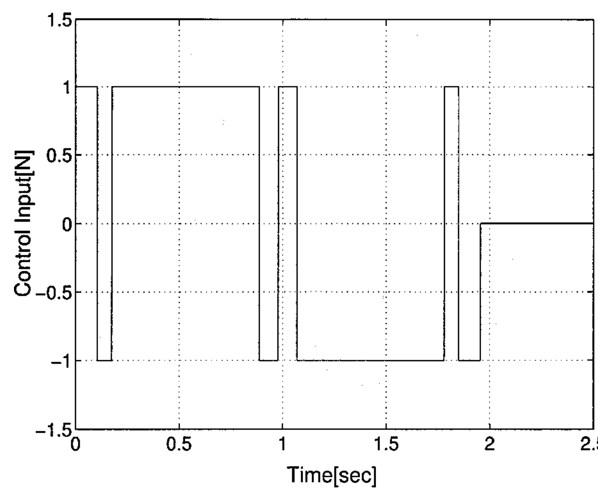

Fig. 5 Time response of control input (robust time-optimal control).

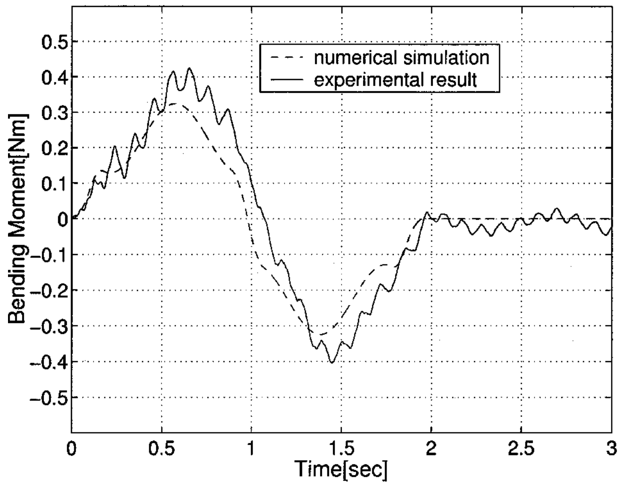

Fig. 6 Time response of bending moment (robust time-optimal control).

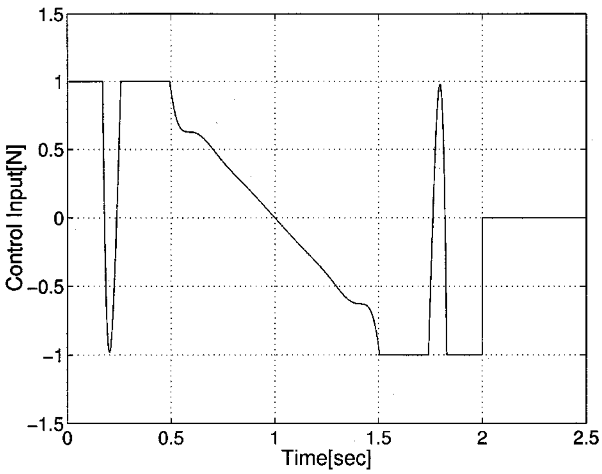

Fig. 7 Time response of control input (MBMC without inequality constraint).

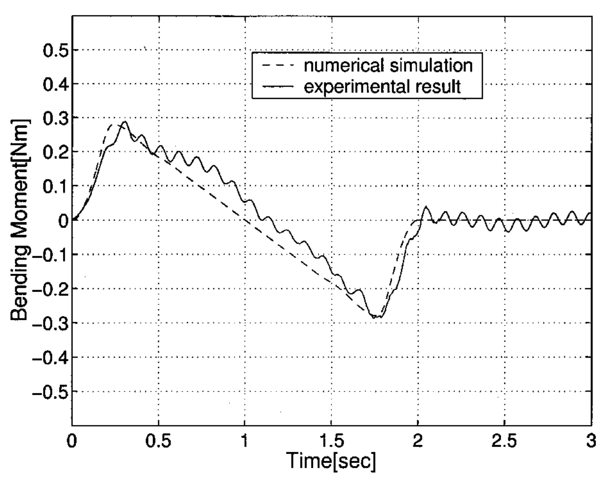

Fig. 8 Time response of bending moment (MBMC without inequality constraints).

is seen to be suppressed by these switchings shortly after the start and before the end of the time response of the control input. This is seen to reduce the inclination of the increase of the bending moment, and this reduction naturally results in the small value of the residual vibration as shown in Fig. 8.

C. Minimum Bending-Moment Control with Inequality Constraint on the Bending Moment

The maximum limit on the bending moment is set to be 0.175 N · m. As the result of numerical analyses, a number of optimal solutions are obtained in accordance with the maneuvering time. The robustness of the solutions is evaluated by

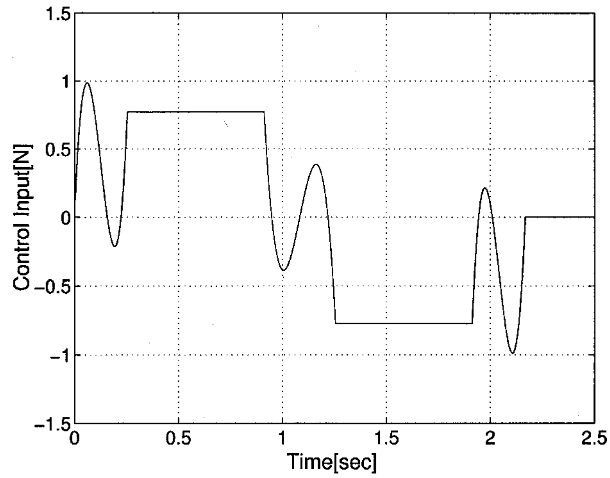

Fig. 9 Time response of control input of MBMC with inequality constraint (minimum maneuvering-time solution).

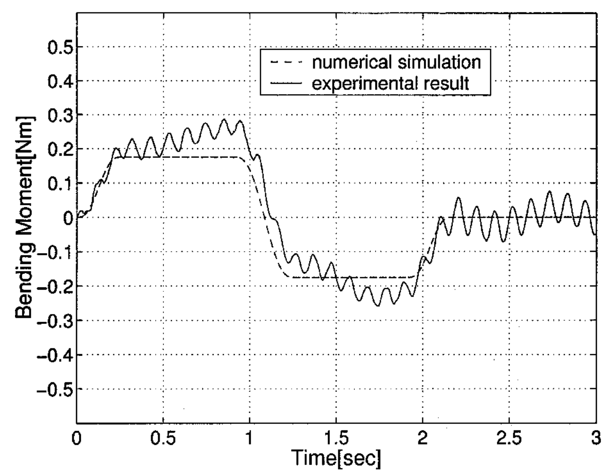

Fig. 10 Time response of bending moment (MBMC with inequality constraint, minimum maneuvering-time solution).

$$R_b := c_1 \left.\frac{\mathrm{d}x_3(t)}{\mathrm{d}\omega}\right|_{t=t_f} + c_2 \left.\frac{\mathrm{d}^2 x_3(t)}{\mathrm{d}\omega^2}\right|_{t=t_f} \tag{33}$$

where c_1 and c_2 are the coefficients to evaluate the first and second derivatives of x_3 with respect to ω, respectively, and are set to be units ($c_1 = c_2 = 1$) in the present study. This value is equivalent to the sum of the first- and second-order sensitivity of the residual vibration energy with respect to the structural frequency uncertainty. It can be expected that the residual vibration will become small even if the structural frequency uncertainty exists when this value is small. Let us discuss two solutions selected among a number of solutions of optimization on the maneuvering time and the robustness: One is the minimum maneuvering-time solution, and the other is a more robust solution or a solution with stronger robustness. The minimum maneuvering time is obtained as 2.17 s, and the final time of the more robust solution is obtained as 2.20 s. The time responses of the control input and the bending moment of the minimum maneuvering-time solution are shown in Figs. 9 and 10, respectively. Figures 11 and 12 show the time responses of the control input and the bending moment at the root of the flexible beam of the more robust solution, respectively. The value of the bending moment is seen to violate the constraint in the experimental results. The difference between the numerical simulations and experiments may be mainly due to the second- and higher-order vibrational modes.

A switching of the control input can be seen at the middle of the maneuvering time in the case of the minimum maneuvering-time

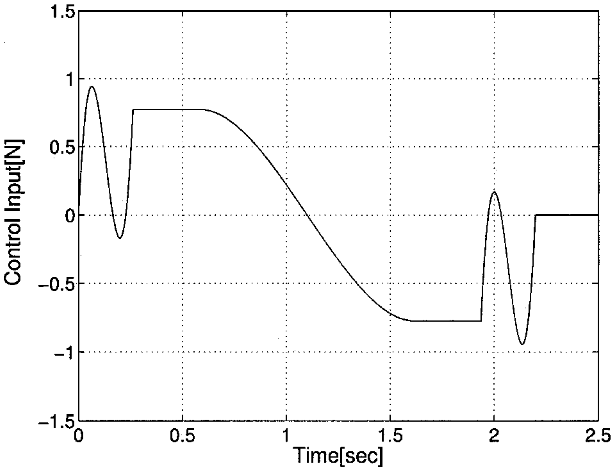

Fig. 11 Time response of control input of MBMC with inequality constraint (solution with stronger robustness).

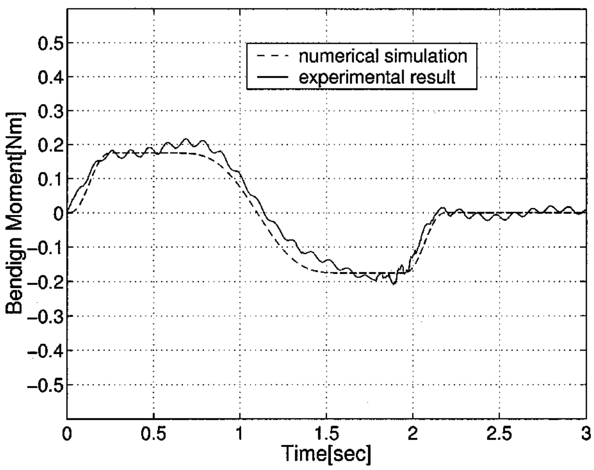

Fig. 12 Time response of bending moment (MBMC with inequality constraint, and stronger robustness).

solution of the minimum bending-moment control with inequality constraint (Fig. 9). A similar switching of the control input is seen at the middle of the maneuvering time in the case of the time-optimal control (Fig. 3), as well as the robust time-optimal control (Fig. 5). On the other hand, no switching of the control input is observed at the middle of the maneuvering time in the case of the more robust solution of the minimum bending-moment control with inequality constraint (Fig. 11). It is understood that the switching of the control input at the middle of the maneuvering time is useful to reduce the maneuvering time for the rest-to-rest maneuver problem of the present flexible space structure, but unfavorable for the robustness of the problem with respect to the structural frequency uncertainty. Another switching of the control input is seen shortly after the start and before the end of maneuver process in the case of the minimum maneuvering-time solution of the minimum bending-moment control with inequality constraint (Fig. 9), even though this solution is chosen to minimize maneuvering time. Similar switchings of control input can also be seen in the more robust solution of the minimum bending-moment control with inequality constraint (Fig. 11). Therefore, it may be said that switching of control input at the begining and the end of the maneuver is helpful to enhance the robustness of the rest-to-rest maneuver problem of the flexible space structure with respect to the structural frequency uncertainty. The control input does not saturate in the case of either the minimum time solution or the more robust solution of the minimum bending-moment control with inequality constraint.

Table 2 shows the summary of numerical simulations employed in the time-optimal control, the robust time-optimal control, and the minimum bending-moment control (MBMC). It is seen that the final time of the MBMC without inequality constraint is longer than that of the time-optimal control by 18% and that of robust time-optimal control by 2%. The value of the performance index in the MBMC without inequality constraint is less than that of the time-optimal control by 51% and that of the robust time-optimal control by 36%. It is also seen that the final times of the minimum maneuvering-time solution and the more robust solution of the MBMC with constraint are longer than that of the time-optimal control by 28 and 29%, respectively, and longer than that of the robust time-optimal control by 11 and 12%, respectively. The values of the performance index in the preceding two solutions are less than that of the time-optimal control by 53 and 59%, respectively, and less than that of the robust time-optimal control by 39 and 47%, respectively. The values of the robustness R_b are also shown in Table 2. The value R_b is not shown in the robust time-optimal control because it is prescribed to be zero, $R_b = 0$, in the analysis. The value of the robustness for the more robust solution is lower than that of the other results. The residual vibration in the experiment with employment of the more robust solution is actually smaller than that of the other results. Thus, it may be fairly said that the MBMC is much more effective in reducing the vibration of the beam, if we take into account the suppression of bending moment at the root of the flexible beam as well as the second-order sensitivity of the residual vibrational energy with respect to the structural frequency uncertainty.

Table 2 Comparison between time-optimal controls and MBMC

Control algorithm	t_f, s	$J = \int_{t_0}^{t_f} M^2(t)\,\mathrm{d}t$	R_b
Time-optimal control	1.70	0.111	0.247
Robust time-optimal control	1.96	0.085	——
MBMC without constraint (fixed final time)	2.00	0.054	0.094
MBMC with constraint, minimum maneuvering-time solution	2.17	0.052	0.099
MBMC with constraint, more robust solution	2.20	0.045	0.068

VI. Conclusions

MBMC is introduced as a problem to minimize the time integral of the square of the bending moment instead of the maneuver time, to reduce the vibration of flexible space structures during slew maneuvers. The problem without a constraint on the maximum value of the bending moment has been converted into a nonlinear programming problem and then solved analytically.

When it was assumed that the time response of the bending moment consists of five phases and the analytical solution was utilized, the problem with the inequality constraints on the maximum value of the bending moment was solved through nonlinear programming. Results of numerical simulations show that the present MBMC can successfully reduce the flexible vibration. To demonstrate the effectiveness of the proposed method, the time response of the control inputs has been implemented in experiments, and the results of numerical analysis and experiment are compared with those of the time-optimal control and the robust time-optimal control.

Successful feasibility of our present analytical formulations is demonstrated through numerical simulations and experiments.

Appendix: Derivation of the Integral Equation

Equation (18) is rewritten as

$$\int_{t_0}^{t_f} \left\{ \nu_1 \phi_0 (t_f - \tau) + \nu_2 \phi_0 + \frac{\nu_3 \phi_1}{\omega} \sin \omega (t_f - \tau) + \nu_4 \phi_1 \cos \omega (t_f - \tau) \right\} \eta(\tau)\,\mathrm{d}\tau + \int_{t_0}^{t_f} \left[\int_{t_0}^{t} \left(\frac{\alpha \phi_1}{\omega} \right)^2 \sin \omega (t - \tau) u(\tau)\,\mathrm{d}\tau \right] \times \left[\int_{t_0}^{t} \sin \omega (t - \tau) \eta(\tau)\,\mathrm{d}\tau \right] \mathrm{d}t = 0 \qquad \text{(A1)}$$

The second term of the left-hand side of Eq. (A1) can be transformed as follows:

$$\int_{t_0}^{t_f}\left[\int_{t_0}^{t}\left(\frac{\alpha\phi_1}{\omega}\right)^2 \sin\omega(t-\tau)u(\tau)\,\mathrm{d}\tau\right]$$

$$\times\left[\int_{t_0}^{t}\sin\omega(t-\tau)\eta(\tau)\,\mathrm{d}\tau\right]\mathrm{d}t$$

$$\leftrightarrow\left(\frac{\alpha\phi_1}{\omega}\right)^2\int_{t_0}^{t_f}\int_{t_0}^{t}\int_{t_0}^{t}\sin\omega(t-s)u(s)\,\mathrm{d}s$$

$$\times\ \sin\omega(t-\tau)\eta(\tau)\,\mathrm{d}\tau\,\mathrm{d}t$$

$$\leftrightarrow\left(\frac{\alpha\phi_1}{\omega}\right)^2\int_{t_0}^{t_f}\int_{\tau}^{t_f}\int_{t_0}^{t}\sin\omega(t-s)$$

$$\times\ \sin\omega(t-\tau)u(s)\,\mathrm{d}s\eta(\tau)\,\mathrm{d}t\,\mathrm{d}\tau$$

$$\leftrightarrow\left(\frac{\alpha\phi_1}{\omega}\right)^2\int_{t_0}^{t_f}\int_{\tau}^{t_f}\int_{t_0}^{t}\sin\omega(t-s)$$

$$\times\ \sin\omega(t-\tau)u(s)\,\mathrm{d}s\,\mathrm{d}t\eta(\tau)\,\mathrm{d}\tau \tag{A2}$$

Furthermore,

$$\int_{\tau}^{t_f}\int_{t_0}^{t}\sin\omega(t-s)\sin\omega(t-\tau)u(s)\,\mathrm{d}s\,\mathrm{d}t$$

$$\leftrightarrow\int_{t_0}^{t_f}\int_{\max(s,\tau)}^{t_f}\sin\omega(t-s)\sin\omega(t-\tau)u(s)\,\mathrm{d}t\,\mathrm{d}s \tag{A3}$$

and employing the integrations

$$Y_1=\int_{\tau}^{t_f}\sin\omega(t-s)\sin(t-\tau)\,\mathrm{d}t \qquad (s<\tau) \tag{A4}$$

$$Y_2=\int_{s}^{t_f}\sin\omega(t-s)\sin(t-\tau)\,\mathrm{d}t \qquad (s>\tau) \tag{A5}$$

Eq. (A3) is written as

$$\int_{t_0}^{t_f}\int_{\max(s,\tau)}^{t_f}\sin\omega(t-s)\sin\omega(t-\tau)u(s)\,\mathrm{d}t\,\mathrm{d}s$$

$$=\int_{t_0}^{\tau}Y_1u(s)\,\mathrm{d}s+\int_{\tau}^{t_f}Y_2u(s)\,\mathrm{d}s$$

$$=\int_{t_0}^{\tau}(Y_1-Y_2)u(s)\,\mathrm{d}s+\int_{t_0}^{t_f}Y_2u(s)\,\mathrm{d}s \tag{A6}$$

Replacing the second term of the right-hand side of Eqs. (A1–A6), we obtain

$$\int_{t_0}^{t_f}\left\{\nu_1\phi_0(t_f-\tau)+\nu_2\phi_0+\frac{\nu_3\phi_1}{\omega}\sin\omega(t_f-\tau)\right.$$

$$+\,\nu_4\phi_1\cos\omega(t_f-\tau)+\int_{t_0}^{\tau}(Y_1-Y_2)\,u(s)\,\mathrm{d}s$$

$$\left.+\int_{t_0}^{t_f}Y_2u(s)\,\mathrm{d}s\right\}\eta(\tau)\,\mathrm{d}\tau=0 \tag{A7}$$

The integral equation (19) is then obtained by rearranging Eq. (A7).

Acknowledgment

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